show that the thermal-expansion coefficients and the piezoelectric coefficients must depend on the stresses. This has been verified experimentally for the former case, but no pertinent experimental results are available to verify or to refute the latter.

Appendix: One-Dimensional Case

The axial stress in a slender, piezothermoelastic bar made of an orthorhombic crystal of class mm2 (Ref. 8) can be expressed as

$$\sigma_1 = Q_{11}\varepsilon_1 - e_{31}E_3 - \lambda_1 \cdot (T - T_0) \tag{A1}$$

where 1 and 3 are, respectively, the axial and thickness coordinates; Q_{11} is the axial elastic stiffness; and e_{31} is the only nonzero piezoelectric coefficient. The thermoelastic coefficient is given by

$$\lambda_1 = Q_{11}\alpha_1 \tag{A2}$$

where α_1 is the axial thermal-expansion coefficient.

Differentiating Eq. (A1) and using transformations similar to those described in the section Analysis of Thermoelastic Coefficients yields

$$\frac{\partial \sigma_1}{\partial T} = F(\sigma_1, E_3, T)\left(\frac{\partial Q_{11}}{\partial T}\right) - E_3\frac{\partial e_{31}}{\partial T} - Q_{11}\frac{\partial[(T - T_0)\alpha_1]}{\partial T} \tag{A3}$$

where

$$F(\sigma_1, E_3, T) \equiv \frac{\sigma_1 + e_{31}E_3}{Q_{11}} \tag{A4}$$

Finally, the counterpart of Eq. (20) is found to be

$$(\alpha_1)(T - T_0) = \int_{T0_0}^{T} \left\{\left[Q_{11}(T_0)\alpha_1(T_0) + F\left(\frac{\partial Q_{11}}{\partial T}\right) - E_3\frac{\partial e_{31}}{\partial T} - \rho T p_3 \frac{\partial E_3}{\partial t}\right] \bigg/ Q_{11}(T)\right\} \mathrm{d}T \tag{A5}$$

Because $F = F(\sigma_1, E_3, T)$, it follows that the axial stress affects the axial thermal-expansion coefficient at temperature different from the reference value provided that the stiffness depends on temperature. It is well known that for typical piezoelectric materials[9] Q_{11} does vary with T.

Also, it is noted that, if $E_3 \neq 0$ and e_{31} depends on T, the thermal-expansion coefficient depends on E_3 as well.

References

[1]Gough, J., "A Description of a Property of Caoutchouc or India-Rubber," *Manchester Philosophical Memoirs*, 2nd Ser., Vol. 1, 1805, pp. 288–295.

[2]Bert, C. W., and Fu, C., "Implications of Stress Dependency of the Thermal Expansion Coefficient on Thermal Buckling," *Journal of Pressure Vessel Technology*, Vol. 114, May 1992, pp. 189–192.

[3]Nye, J. F., *Physical Properties of Crystals*, Clarendon, Oxford, England, UK, 1964.

[4]Parton, V. Z., and Kudryavtsev, B. A., *Electromagnetoelasticity: Piezoelectrics and Electrically Conductive Solids*, Gordon and Breach, New York, 1988, Chap. 1.

[5]Rosenfield, A. R., and Averbach, B. L., "Effect of Stress on the Expansion Coefficient," *Journal of Applied Physics*, Vol. 27, Feb. 1956, pp. 154–156.

[6]Lee, H.-J., and Saravanos, D. A., "Coupled Layerwise Analysis of Thermopiezoelectric Composite Beams," *AIAA Journal*, Vol. 34, No. 6, 1996, pp. 1231–1237.

[7]Krueger, H. H. A., "Stress Sensitivity of Piezoelectric Ceramics: Part 3, Sensitivity to Compressive Stress Perpendicular to the Polar Axis 3," *Journal of the Acoustical Society of America*, Vol. 43, No. 3, 1968, pp. 583–591.

[8]Jonnalagadda, K. D., Blandford, G. E., and Tauchert, T. R., "Piezothermoelastic Composite Plate Analysis Using First-Order Shear Deformation Theory," *Computers and Structures*, Vol. 51, No. 1, 1994, pp. 79–89.

[9]*Guide to Modern Piezoelectric Ceramics*, Morgan Matroc, Inc., Bedford, OH, 1993, p. 10.

G. A. Kardomateas
Associate Editor

Parametric Resonance of Cylindrical Shells by Different Shell Theories

K. Y. Lam* and T. Y. Ng†
National University of Singapore, Singapore 119260, Republic of Singapore

I. Introduction

THE dynamic stability of thin, isotropic cylindrical shells under combined static and periodic axial forces is studied using three common thin-shell theories, namely, Donnell's,[1] Love's,[2] and Flügge's[3] shell theories. A main feature of this work is that, for each shell theory, the contribution of the stresses due to the external forces is accounted for according to the assumptions made in that particular shell theory. This is an extension of Ref. 4, in which consideration for the axial loading was based only on Donnell's[1] theory.

Studies of buckling of thin-walled isotropic cylinders under axial compression, torsional loadings, bending, hydrostatic pressure, and lateral pressure have been extensively covered. However, structural components under periodic loads can undergo parametric resonance that may occur over a range of forcing frequencies, and if the load is compressive to the structure, resonance or instability can and usually does occur even if the magnitude of the load is below the critical buckling load of the structure. It is thus of prime importance to investigate the dynamic stability of dynamic systems under periodic loads. The parametric resonance of cylindrical shells under axial loads has become a popular subject of study. It was first examined by Bolotin,[5] Yao,[6] and Vijayaraghavan and Evan-Iwanowski.[7] For thin cylindrical shells under periodic axial loads, the method of solution is almost always first to reduce the equations of motion to a system of Mathieu–Hill equations. The dynamic stability for such a system of equations can then be analyzed by a number of methods.

In the present analysis, the dynamic stability of thin, isotropic cylindrical shells under combined static and periodic axial forces is studied using three different shell theories: Donnell's,[1] Love's,[2] and Flügge's.[3] The treatment of the stresses due to the external loadings is done based on the assumptions made in that shell theory. A normal-mode expansion yields a system of Mathieu–Hill equations, and the parametric resonance response is analyzed based on Bolotin's[5] method. The present formulation of the problem is also made general to accommodate any boundary conditions, but for reasons of simplicity, the comparison study is carried out only for the case of simply supported boundary conditions. Numerical results of the instability regions are presented and are compared with those of Ref. 4.

II. Theory and Formulation

The cylindrical shell considered is as in Ref. 4, a thin, uniform shell of length L, thickness h, and radius R. The extensional pulsating axial load is given by

$$N_a(x, t) = N_o + N_s \cos Pt \tag{1}$$

where P is the frequency of excitation in radians per unit time.

In the present analysis, three shell theories for a thin-walled cylindrical shell are compared. They are Donnell's,[1] Love's,[2] and Flügge's[3] theories for thin cylindrical shells. The theoretical formulation follows that of Ref. 4, but the consideration for the dynamic

Received Aug. 20, 1997; revision received July 1, 1998; accepted for publication Aug. 24, 1998.

*Associate Professor, Department of Mechanical and Production Engineering, 10 Kent Ridge Crescent.

†Research Scholar, Department of Mechanical and Production Engineering, 10 Kent Ridge Crescent.

axial loading is based on the assumptions of each of the shell theories considered. In Ref. 4, the dynamic axial loading was considered based on Donnell's[1] theory for all of the shell theories examined.

Thus, the unified governing equations of motion for the dynamic stability analysis of a thin cylindrical shell under pulsating axial load N_a is given by

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\theta x}}{\partial \theta} - \psi_1 \frac{1}{R}\frac{\partial M_{\theta x}}{\partial \theta} + \zeta_1 \frac{\partial}{\partial x}\left(N_a \frac{\partial u}{\partial x}\right) = \rho_t \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \psi_2 \frac{1}{R}\frac{\partial M_{x\theta}}{\partial x} + \psi_3 \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} + \zeta_2 \frac{\partial}{\partial x}\left(N_a \frac{\partial v}{\partial x}\right)$$

$$= \rho_t \frac{\partial^2 v}{\partial t^2} \tag{2}$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{1}{R}\left(\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{\partial^2 M_{\theta x}}{\partial x \partial \theta}\right) + \frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{1}{R} N_\theta$$

$$+ \zeta_3 \frac{\partial}{\partial x}\left(N_a \frac{\partial w}{\partial x}\right) = \rho_t \frac{\partial^2 w}{\partial t^2}$$

where ψ_i $(i = 1, 2, 3)$ are tracers used to unify the equations of motion for the three shell theories used. The tracers ζ_i $(i = 1, 2, 3)$ are used to unify the equations of motion, where the treatment for the pulsating axial load N_a varies for the three shell theories. The values of the tracers for Donnell's[1] theory are $\psi_1, \psi_2, \psi_3, \zeta_1, \zeta_2, \zeta_3 = 0, 0, 0, 1, 0, 0$. For Love's[2] theory, $\psi_1, \psi_2, \psi_3, \zeta_1, \zeta_2, \zeta_3 = 0, 1, 1, 1, 1, 0$, and for Flügge's[3] theory, $\psi_1, \psi_2, \psi_3, \zeta_1, \zeta_2, \zeta_3 = 0, 1, 1, 1, 1, 1$.

The equations of motion by Love's[2] theory differ from those of Flügge's[3] in that the force component due to N_a for the equilibrium along the u direction is neglected. The equations of motion by Donnell's[1] theory assume that the shear component in the θ direction $[\partial M_{x\theta}/\partial x + 1/a(\partial M_\theta/\partial\theta)]$ can be neglected. They also assume that the curvature v along the x direction can be neglected.

The following nondimensionalized parameters are introduced to simplify the formulation:

$$\eta_s = \frac{N_s(1-\nu^2)}{Eh}, \qquad \eta_o = \frac{N_o(1-\nu^2)}{Eh} \tag{3}$$

$$\bar{t} = t\left[\frac{E}{\rho R^2(1-\nu^2)}\right]^{\frac{1}{2}} \tag{4}$$

$$p = P\left[\frac{\rho R^2(1-\nu^2)}{E}\right]^{\frac{1}{2}}, \qquad \bar{\omega} = \omega\left[\frac{\rho R^2(1-\nu^2)}{E}\right]^{\frac{1}{2}} \tag{5}$$

where ω is the natural frequency of the cylindrical shell under the constant axial load N_o, with the oscillating component $N_s = 0$.

If the shell assumed is simply supported, there exists a solution for the equations of motion given by the form

$$u_{mn} = A_{mn}\, e^{i\bar{\omega}\bar{t}} \cos(m\pi x/L)\cos n\theta \tag{6}$$

$$v_{mn} = B_{mn}\, e^{i\bar{\omega}\bar{t}} \sin(m\pi x/L)\sin n\theta \tag{7}$$

$$w_{mn} = C_{mn}\, e^{i\bar{\omega}\bar{t}} \sin(m\pi x/L)\cos n\theta \tag{8}$$

where n represents the number of circumferential waves and m represents the number of axial half-waves in the corresponding standing-wave pattern.

The equations of motion can be solved by using an eigenfunction expansion in terms of the normal modes of the free vibrations of a cylindrical shell under a constant axial load N_o with the oscillating component $N_s = 0$. Substitution of Eqs. (6–8) into the equations of motion, which are a set of three coupled homogeneous equations, yields a cubic frequency equation when the determinant is equated to zero. Thus, for each m and n, there exist three roots corresponding to the transverse, axial, and circumferential modes.

To solve the equations of motion that include the oscillating component N_s, a solution is sought in the following form, where all of the modes are superimposed:

$$u_{mnj} = \sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mnj}\bar{q}_{mnj}(\bar{t})\cos\lambda_m x\cos n\theta \tag{9}$$

$$v_{mnj} = \sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mnj}\bar{q}_{mnj}(\bar{t})\sin\lambda_m x\sin n\theta \tag{10}$$

$$w_{mnj} = \sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mnj}\bar{q}_{mnj}(\bar{t})\sin\lambda_m x\cos n\theta \tag{11}$$

where $\bar{q}_{mnj}(\bar{t})$ is a generalized coordinate and

$$\lambda_m = m\pi/L \tag{12}$$

Thus, substituting Eqs. (9–11) into the equations of motion (2) and simplifying yields

$$\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\ddot{\bar{q}}_{mnj} + \bar{\omega}_{mnj}^2\bar{q}_{mnj}\right)\Gamma_{mnj}\cos\lambda_m x\cos n\theta$$

$$+ \zeta_1 R^2\lambda_m\cos p\bar{t}\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Gamma_{mnj}\bar{q}_{mnj}\frac{\partial}{\partial x}$$

$$\times(\eta_s\sin\lambda_m x)\cos n\theta = 0 \tag{13}$$

$$\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\ddot{\bar{q}}_{mnj} + \bar{\omega}_{mnj}^2\bar{q}_{mnj}\right)\beta_{mnj}\sin\lambda_m x\sin n\theta$$

$$- \zeta_2 R^2\lambda_m\cos p\bar{t}\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_{mnj}\bar{q}_{mnj}\frac{\partial}{\partial x}$$

$$\times(\eta_s\cos\lambda_m)\sin n\theta = 0 \tag{14}$$

$$\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\ddot{\bar{q}}_{mnj} + \bar{\omega}_{mnj}^2\bar{q}_{mnj}\right)\sin\lambda_m x\cos n\theta$$

$$- \zeta_3 R^2\lambda_m\cos p\bar{t}\sum_{j=1}^{3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\bar{q}_{mnj}\frac{\partial}{\partial x}$$

$$\times(\eta_s\cos\lambda_m x)\cos n\theta = 0 \tag{15}$$

where

$$\Gamma_{mnj} = A_{mnj}/C_{mnj} \tag{16}$$

$$\beta_{mnj} = B_{mnj}/C_{mnj} \tag{17}$$

Making use of the orthogonality condition, we multiply Eq. (13) by $\Gamma_{rsi}\cos\lambda_r x\cos s\theta$, Eq. (14) by $\beta_{rsi}\sin\lambda_r x\sin s\theta$, and Eq. (15) by $\sin\lambda_r x\cos s\theta$. We then add the three resulting equations and integrate over the surface of the cylinder. This yields the following set of equations:

$$\boldsymbol{M}_{IJ}\ddot{\bar{\boldsymbol{q}}}_J + (\boldsymbol{K}_{IJ} - \cos p\bar{t}\boldsymbol{Q}_{IJ})\bar{\boldsymbol{q}}_J = 0 \tag{18}$$

where $\boldsymbol{M}_{IJ}$, $\boldsymbol{K}_{IJ}$, and $\boldsymbol{Q}_{IJ}$ are matrices and $\ddot{\bar{\boldsymbol{q}}}_J$ and $\bar{\boldsymbol{q}}_J$ are column vectors consisting of $\ddot{\bar{q}}_{mnj}$ and $\bar{q}_{mnj}$.

The subscripts r, s, i, m, n, j, I, and J have the following ranges: $i, j = 1, 2, 3$; $r, s, m, n = 1, 2, \ldots, N$; and $I, J = 1, 2, \ldots, 3N^2$.

The matrices $\boldsymbol{M}_{IJ}$, $\boldsymbol{K}_{IJ}$, and $\boldsymbol{Q}_{IJ}$ are given as

$$\boldsymbol{M}_{IJ} = \int_0^L\int_0^{2\pi}(\Gamma_I\Gamma_J\cos\lambda_r x\cos s\theta\cos\lambda_m x\cos n\theta$$

$$+ \beta_I\beta_J\sin\lambda_r x\sin s\theta\sin\lambda_m x\sin n\theta$$

$$+ \sin\lambda_r x\cos s\theta\sin\lambda_m x\cos n\theta)\,\mathrm{d}\theta\,\mathrm{d}x$$

$$= \begin{cases}\frac{1}{2}\pi L(1 + \Gamma_I\Gamma_J + \beta_I\beta_J) & \text{if} \quad I = J\\ 0 & \text{if} \quad I \neq J\end{cases} \tag{19}$$

$$\boldsymbol{K}_{IJ} = \boldsymbol{M}_{IJ}\bar{\omega}_J^2 \tag{20}$$

$$\begin{aligned}\boldsymbol{Q}_{IJ} &= -\zeta_1 \Gamma_I \Gamma_J \lambda_m \int_0^L \int_0^{2\pi} \frac{\partial}{\partial x}(\eta_s \sin\lambda_m x \cos n\theta) \\ &\times \cos\lambda_r x \cos s\theta \,\mathrm{d}\theta\,\mathrm{d}x + \zeta_2 \beta_I \beta_J \lambda_m \int_0^L \int_0^{2\pi} \frac{\partial}{\partial x} \\ &\times (\eta_s \cos\lambda_m x \sin n\theta) \sin\lambda_r x \sin s\theta \,\mathrm{d}\theta\,\mathrm{d}x \\ &+ \zeta_3 \lambda_m \int_0^L \int_0^{2\pi} \frac{\partial}{\partial x}(\eta_s \cos\lambda_m x \cos n\theta) \sin\lambda_r x \cos s\theta \,\mathrm{d}\theta\,\mathrm{d}x \\ &= \begin{cases} -(\zeta_1 \Gamma_I \Gamma_J + \zeta_2 \beta_I \beta_J + \zeta_3) R^2 \frac{1}{2}\pi L \lambda_r \lambda_m \eta_s & \text{if} \quad I = J \\ 0 & \text{if} \quad I \neq J \end{cases}\end{aligned} \tag{21}$$

III. Stability Analysis

Equation (18) is in the form of a second-order differential equation with periodic coefficients of the Mathieu–Hill type. The dynamic stability analysis is based on Bolotin's[5] method and can be found in Ref. 4.

IV. Numerical Results and Discussion

For the present results, Poisson's ratio ν is taken to be 0.3. Each unstable region is bounded by two curves originating from a common point from the p axis with $\eta_s = 0$. For the sake of tabular presentation, the angle subtended, Θ, is introduced. Its definition is given in Ref. 4.

For periodic compressive loads, the compressive axial loads should not exceed the critical buckling load η_{cr} of the cylindrical shell. For cylindrical shells of intermediate length, as in the cases used here, the buckling load given by Timoshenko and Gere,[8] which is also used in Ref. 4, is

$$\eta_{\mathrm{cr}} = P_{\mathrm{cr}}[(1 - \nu^2)/Eh] \tag{22}$$

The effects of variation of the length ratios L/R and the thickness ratios R/h for the fundamental instability region of a cylindrical shell under a tensile loading of $\eta_o = 0.1\eta_{\mathrm{cr}}$ are shown in Tables 1–3. The results are compared with those of Ref. 4, in which loading conditions were modeled according to assumptions made in Donnell's[1] theory for all of the different shell theories used.

Both sets of results show similar trends in terms of dynamic behavior. The points of origin of the unstable regions are lower for the longer and thinner shells. Also, the sizes of the fundamental unstable regions decrease with increased cylinder length. Both sets

Table 1 Unstable regions associated with the fundamental modes for a shell under tensile loading of $\eta_o = 0.1\eta_{\mathrm{cr}}$ at various R/h ratios and $L/R = 2$

	$R/h = 100$, fundamental mode (1, 5)		$R/h = 110$, fundamental mode (1, 5)		$R/h = 120$, fundamental mode (1, 5)	
Theory	Present	Reference 4	Present	Reference 4	Present	Reference 4
Donnell[1]						
$p\ (\times 10^{-1})$	2.3966085	2.3966085	2.2974519	2.2974519	2.2182939	2.2182939
$\Theta\ (\times 10^{-3})$	5.4034474	5.4034474	5.1245751	5.1245751	4.8656016	4.8656016
Love[2]						
$p\ (\times 10^{-1})$	2.3644021	2.3586483	2.2700992	2.2647545	2.1948460	2.1898602
$\Theta\ (\times 10^{-3})$	5.7014782	5.4893049	5.3989275	5.1976332	5.1192753	4.9280201
Flügge[3]						
$p\ (\times 10^{-1})$	2.3716829	2.3668536	2.2763895	2.2718157	2.2003332	2.1959959
$\Theta\ (\times 10^{-3})$	5.6980983	5.4706654	5.3973476	5.1818136	5.1191553	4.9145404

Table 2 Unstable regions associated with the fundamental modes for a shell under tensile loading of $\eta_o = 0.1\eta_{\mathrm{cr}}$ at various R/h ratios and $L/R = 5$

	$R/h = 100$, fundamental mode (1, 3)		$R/h = 110$, fundamental mode (1, 3)		$R/h = 120$, fundamental mode (1, 3)	
Theory	Present	Reference 4	Present	Reference 4	Present	Reference 4
Donnell[1]						
$p\ (\times 10^{-1})$	0.9572622	0.9572622	0.9292900	0.9292900	0.9071538	0.9071538
$\Theta\ (\times 10^{-3})$	2.0227872	2.0227872	1.8946057	1.8946057	1.7794501	1.7794501
Love[2]						
$p\ (\times 10^{-1})$	0.9336408	0.9284480	0.9096135	0.9048069	0.8862943	0.8835154
$\Theta\ (\times 10^{-3})$	2.3072559	2.0848530	2.1534687	1.9453175	2.0309892	1.9165997
Flügge[3]						
$p\ (\times 10^{-1})$	0.9387206	0.9338362	0.9139379	0.9093776	0.8917600	0.8892861
$\Theta\ (\times 10^{-3})$	2.3031299	2.0730090	2.1510647	1.9356816	2.0212832	1.9043277

Table 3 Unstable regions associated with the fundamental modes for a shell under tensile loading of $\eta_o = 0.1\eta_{\mathrm{cr}}$ at various R/h ratios and $L/R = 10$

	$R/h = 100$, fundamental mode (1, 6)		$R/h = 110$, fundamental mode (1, 6)		$R/h = 120$, fundamental mode (1, 6)	
Theory	Present	Reference 4	Present	Reference 4	Present	Reference 4
Donnell[1]						
$p\ (\times 10^{-2})$	4.7899298	4.7899298	4.6919409	4.6919409	4.6147815	4.6147815
$\Theta\ (\times 10^{-3})$	0.8976778	0.8976778	0.8333198	0.8333198	0.7768298	0.7768298
Love[2]						
$p\ (\times 10^{-2})$	4.6334781	4.5839896	4.5642140	4.5187263	4.3922142	4.3697829
$\Theta\ (\times 10^{-3})$	1.1612415	0.9376017	1.0721156	0.8649618	1.0247256	0.9268837
Flügge[3]						
$p\ (\times 10^{-2})$	4.6660968	4.6179227	4.5916810	4.5471908	4.4338792	4.4130819
$\Theta\ (\times 10^{-3})$	1.1582715	0.9307977	1.0704416	0.8596118	0.9705777	0.9178957

of results also show that the three shell theories, with the exception of Donnell's[1] theory, agree well with each other, in terms of both point of origin and size. This trend regarding the relative accuracies between shell theories was also observed and reported by Lam and Loy[9] in the free-vibration analysis of rotating laminated cylindrical shells. It is also observed that, as the length ratio L/R increased, the agreement between Donnell's[1] theory and the other two theories deteriorates. This too was noted by Lam and Loy[9] in the free-vibration analysis.

In the present results, for relatively shorter shells of length ratio $L/R = 2$, Donnell's[1] theory gives results in terms of the instability region size that are 5–6% higher or less conservative than those of the other two theories. For shells of $L/R = 5$, the corresponding results are 13–14% less conservative, and for relatively longer shells of $L/R = 10$, the corresponding results are 29–31% less conservative. The major difference between the present results and those of Ref. 4 is that the present results are considerably more conservative, in terms of instability region size, for Love's[2] and Flügge's[3] theories. For shorter shells of length ratio $L/R = 2$, the present results for Love's and Flügge's theories are about 4% more conservative than those of Ref. 4. For shells of $L/R = 5$, the present results are about 11% more conservative, and for relatively longer shells of $L/R = 10$, the present results are 24% more conservative. The present results also indicate that Love's theory generates the most conservative results.

V. Conclusion

Three shell theories—Donnell's,[1] Love's,[2] and Flügge's[3]—were used in the dynamic stability analysis of simply supported, thin, isotropic cylindrical shells under combined static and periodic axial forces, where a system of Mathieu–Hill equations was obtained via a normal-mode expansion of the equations of motion, and the parametric resonance response was analyzed by using Bolotin's[5] method. The main feature of this work is that, for each shell theory considered, the contribution of the stresses due to the external forces is accounted for according to the assumptions made in that theory. The results showed that this consideration yielded considerably more conservative results for Love's and Flügge's theories when compared with corresponding existing results,[4] where consideration for the axial loading was based on Donnell's theory for all of the different theories used.

References

[1] Donnell, L. H., "Stability of Thin Walled Tubes Under Torsion," NACA Rept. 479, 1933.

[2] Love, A. E. H., *A Treatise on the Mathematical Theory of Elasticity*, 4th ed., Cambridge Univ. Press, New York, 1952, Chap. 24.

[3] Flügge, W., *Stresses in Shells*, Springer–Verlag, Berlin, 1933, Chap. 5.

[4] Lam, K. Y., and Ng, T. Y., "Dynamic Stability Analysis of Cylindrical Shells Subjected to Conservative Periodic Axial Loads Using Different Thin Shell Theories," *Journal of Sound and Vibration*, Vol. 207, No. 4, 1997, pp. 497–520.

[5] Bolotin, V. V., *The Dynamic Stability of Elastic Systems*, Holden–Day, San Francisco, 1964, Chap. 14.

[6] Yao, J. C., "Nonlinear Elastic Buckling and Parametric Excitation of a Cylinder Under Axial Loads," *Journal of Applied Mechanics*, Vol. 29, March 1965, pp. 109–115.

[7] Vijayaraghavan, A., and Evan-Iwanowski, R. M., "Parametric Instability of Circular Cylindrical Shells," *Journal of Applied Mechanics*, Vol. 31, Dec. 1967, pp. 985–990.

[8] Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, McGraw–Hill, New York, 1961, Chap. 11.

[9] Lam, K. Y., and Loy, C. T., "Analysis of Rotating Laminated Cylindrical Shells Using Different Shell Theories," *Journal of Sound and Vibration*, Vol. 186, No. 1, 1995, pp. 23–35.

G. A. Kardomateas
Associate Editor